

A37228

Calculators may be used in this examination provided they are not capable of being used to store alphabetical information other than hexadecimal numbers



**UNIVERSITY OF
BIRMINGHAM**

School of Computer Science

First Year Undergraduate

06-34238

LC Artificial Intelligence 1

Main Summer Examinations 2025

Time allowed: 2 hours

[Answer all questions]

Note

Answer ALL questions.

Question 1 Supervised Learning

- (a) The following pseudo-code represents one iteration of passing through the training set for gradient descent applied to univariate logistic regression. It classifies a univariate input x_j into either class label 1 or class label 0 (i.e. $y_j \in \{0, 1\}$).

```

cost = 0;
w0 = 0;
w1 = 0;
for j in size(trainingSet)
    f =  $\frac{1}{1+e^{-(w_0+w_1*x_j)}}$ 
    cost = cost + g
    w0 = w0 + a * (yj - f)
    w1 = w1 + a * (yj - f) * xj
endfor

```

Assume that the value of the learning rate, a is 1. The cost function g is:

$$g = -y_j \ln f - (1 - y_j) \ln (1 - f)$$

- (i) Calculate the numerical values of 'cost', ' w_0 ' and ' w_1 ' at the end of this pseudo-code passing through the following training set once: $\{(1, 1), (-1, 0)\}$, given in (x_j, y_j) pairs. Note that $\ln 0.5 = -0.69$. Show all your working. **[18 marks]**
- (ii) For the final model obtained from question (i), what is the label y for the new input $x = 2$? Justify your answer. **[5 marks]**
- (b) Please answer the following questions about supervised learning:
- Explain how you would find out whether a model is overfitting or underfitting its training data?
 - Given a supervised learning problem, assume that you have obtained several models based on the same training dataset. Half are overfitting, and the other half are underfitting, but you do not know which. You cannot see or change the training data. Can you propose a solution to get better generalization performance?

[12 marks]

Question 2 Unsupervised Learning

(a) This question has two parts. Please answer both.

- Let $c_1 = (4, 3)$, $c_2 = (1, 1)$ and $c_3 = (2, 5)$ represent the centroids of clusters 1, 2 and 3 respectively. Using squared Euclidean distance as the proximity measure, assign the data points (p_1 to p_5) in Table 1 to one of the three clusters. Write down the newly obtained clusters with all your working. Then

Label	Data
p_1	(2, 2)
p_2	(3, 5)
p_3	(3, 3)
p_4	(4, 1)
p_5	(1, 5)

Table 1: Data Set

answer which of these clusters has the highest inertia?

Hint: The inertia of a cluster C is defined as the sum of the squared Euclidean distances from each example e in cluster C to its centroid, i.e., $\text{inertia}(C) = \sum_{e \in C} d_{\text{Euc}}^2(e, \text{centroid}(C))$.

[15 marks]

- Is it possible that the k-means algorithm re-visits an earlier cluster assignment in later iterations before it converges? Explain your answer.

[5 marks]

(b) In Figure 1 below, every square dot in the shape denotes a data point. Which linkage – single or complete – will group the data points in Figure 1 into two clusters (round and diamond dots) as shown in Figure 2? Assume that hierarchical clustering is run until you obtain two clusters. Explain your answer.



Figure 1: Original Data



Figure 2: Required Clusters

[10 marks]

Question 3 Search & Optimisation

(a) (Drone Delivery Optimisation) A single autonomous delivery drone transports N packages, aiming to minimise the total number of trips while satisfying weight, distance, and energy constraints. Each package i has a weight w_i and a delivery distance d_i , $i = 1, \dots, N$. The drone needs to operate whilst ensuring some conditions: (i) The drone can carry at most 10 kg per trip. (ii) The drone can travel at most 50 km per trip (round trip). (iii) Each trip consumes energy E_t , defined by an energy cost function g that depends on the farthest delivery distance in trip t and the total weight of packages in trip t . (iv) The drone has a maximum energy capacity E_{max} per trip.

- Define the design variables and the objective function. **[7 marks]**
- Formalise and state the constraints using the design variables. **[10 marks]**
- Using the design variables, objective function, and the constraints, write down the corresponding optimisation problem in the canonical form. **[3 marks]**
- Write down a suitable energy cost function g , i.e., suitable expression for E_t , for this drone according to the specifications in the description above. Please explain your answer. **[2 marks]**

Hints: The drone can carry multiple packages in each trip and each package has to be delivered once. Assume that the one-way travel distance for each trip is determined by the farthest package delivery in that trip.

(b) (Drone Grid Search) Suppose this delivery drone can navigate a 2-D grid from a start position S_0 to a goal position S_g . The drone can only move in four directions (north, south, west, and east) but cannot pass through obstacles. Each move has a unitary cost.

- Formally define this search problem, specifying initial state, action set, goal test, transition model, and the path cost function. **[10 marks]**
- Suppose we use a heuristic function based on the Manhattan distance to the goal. Explain how A^* search would use this heuristic. **[3 marks]**
The Manhattan distance to the goal is given by:

$$distance((x_1, y_1), (x_2, y_2)) = |x_1 - x_2| + |y_1 - y_2|$$

where (x_1, y_1) , (x_2, y_2) are the state of the current node and the goal state, respectively, and the operation $|k|$ denotes the absolute value of k .

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Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

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- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) **must** be placed in the designated area.
- Check that you **do not** have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are **not** permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
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